INSTRUCTIONS FOR THE STUDENTS

1. Students should solve the Assignment on A4 Size Paper.
2. Four Questions are to be attempted by selecting one question from each unit. All questions carry equal marks.
3. Students are required to submit the solved Assignment(s) either by post or in person in the Directorate of Distance Education, M.D. University, Rohtak by 28.02.2021.
4. The student should fill his/her particulars in the following format on first page of solved Assignment:

Name of the Programme ___________ Nomenclature of the Paper _________________

Paper Code: _____________________ Academic Session __________________________

Student ID: ___________________ Name of Student __________________________

Date of Submission of Solved Assignment ____

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Signature of the Student

Master of Science (Mathematics)
Abstract Algebra
(SEMESTER-1)
Paper Code-20MAT21C1

Attempt 4 questions in all

Maximum Marks : 20

1. If H is a Sylow p-subgroup of G, then prove that $x^{-1}Hx$ is also a sylow p-subgroup of G for any $x \in G$.

-OR-

Show that a Sylow p-subgroup of a finite group G is unique iff it is normal.

2. **Prove that** subgroup of a nilpotent group is nilpotent.

-OR-

State and prove **Scherier's Refinement Theorem.**

3. Let R be a commutative ring with unity and e(e ≠ 0, e ≠ 1) be an idempotent in R. Let M = Re, then M is not a free module.

-OR -

Let N be an R–module and let $End_R(M)$ be the set of all R–homomorphisms of N into N. Make appropriate definitions of addition and multiplication of element of so that $End_R(M)$ become a ring.

4. Let R be a left Noetherian ring and $x_1, x_2, ..., x_n$ be n independent indeterminants. Then, prove that $R[x_1, x_2, ..., x_n]$ is also Noetherian.

-OR-

Let R be a left Artinian ring with unity having no nonzero nilpotent ideals. Prove that every nonzero left ideal of R contains nonzero idempotents.

Master of Science (Mathematics)
Mathematical Analysis

(SEMESTER-1)
Paper Code-20MAT21C2

Attempt 4 questions in all

Maximum Marks : 20

1. **If P^* is a refinement of P, then**

(i) $L(P^*, f, \alpha) \geq L(P, f, \alpha)$

(ii) $U(P^*, f, \alpha) \leq U(P, f, \alpha)$

-OR-

Evaluate the following :

(i) $\int_0^2 x^2 \, dx^2$ **(ii)** $\int_0^2 [x] \, dx^2$

2. **State and prove Cauchy's criterion for uniform convergence on sequence.**

-OR-

Show that the sequence $< f_n >$ where $f_n(x) = \dfrac{x}{1+nx^2}$ converges uniformly on R.

3. Find the radius of convergence of the series :

(i) $x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \ldots\ldots$ (ii) $1 + x + 2!x^2 + 3!x^3 + \ldots\ldots$

-OR -

State and prove **Young's theorem.**

4. Expand $x^2y + 3y - 2$ in powers of ($x-1$) and ($y-2$).

-OR-

Prove that by the transformation $u = x - ct$, $v = x + ct$, the partial diff. equation $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$ reduces to $\dfrac{\partial^2 y}{\partial u \partial v} = 0$.

Master of Science (Mathematics)
Ordinary Differential Equations
(SEMESTER-1)
Paper Code-20MAT21C3

Attempt 4 questions in all

Maximum Marks : 20

1. State and prove Ascoli Lemma.

-OR-

State and prove Picard-Lindelof Theorem.

2. Prove that the *A* necessary and sufficient condition that a solution matrix $\Phi(t)$ of $X'(t) = A(t)$ $X(t)$ to be a fundamental matrix of (LH) is that det $\{\Phi(t)\} \neq 0$ for all $t \in I$.

-OR-

Verify the Sturm's fundamental comparison theorem in the case of real solutions of the differential equations $\frac{d^2u}{dt^2} + A^2u = 0$ and $\frac{d^2u}{dt^2} + B^2u = 0$ where *A* and *B* are constants such that $B > A > 0$.

3. Determine the nature of the critical point (0**,** 0) of the system $\left.\begin{aligned} \frac{dx}{dt} &= 2x - 7y \\ \frac{dy}{dt} &= 3x - 8y \end{aligned}\right]$ and determine whether or not the point is stable.

-OR -

Explain the Critical points and paths of nonlinear systems.

4. Find the nature and stability of the critical point (0, 0) for the non-linear system

$$\left.\begin{aligned} \frac{dx}{dt} &= x + 4y - x^2 \\ \frac{dy}{dt} &= 6x - y + 2xy \end{aligned}\right\}$$

Also, find the family of the paths of this system.

-OR-

Prove that the eigen values of the Sturm Liouville boundary value problem are always real.

Master of Science (Mathematics)
Complex Analysis

(SEMESTER-1)
Paper Code-20MAT21C4

Maximum Marks : 20

Attempt 4 questions in all

1. Express the Cauchy-Riemann equations in polar form.

-OR-

Find the radius of convergence of following power series :

(i) $\sum \frac{z^n}{n^n}$ **(ii)** $\sum \frac{2^{-n} z^n}{1 + i n^2}$

2. State and prove **Cauchy Integral Formula**

-OR-

State and prove **Liouville's theorem.**

3. **Prove that** if an analytic function $f(z)$ has a pole of order m at $z = a$, then $1/f(z)$ has a zero of order m at $z = a$ and conversely.

-OR -

State and prove **Fundamental theorem of algebra**

4. State and prove **Cauchy Residue Theorem.**

-OR-

State and prove **Hurwitz's Theorem**

Master of Science (Mathematics)
Mathematical Statistics
(SEMESTER-1)
Paper Code-20MAT21C5

Maximum Marks : 20

Attempt 4 questions in all

1. A bag contains three coins, one of which is a coin with two heads while the other two coins are normal and not biased. A coin is chosen at random from the bag and tossed four times in succession. If head turns up each time, what is the probability that this is the two –headed coin?

-OR-

State and prove Baye's Theorem.

2. A two-dimensional random variable (X, Y) have a joint probability mass function: $p(x, y) = \frac{1}{27}(x + 2y)$, for x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for $X = x$.

-OR-

State and prove the addition theorem of expectation for two discrete random variables X and Y.

3. Obtain mean and variance of binomial distribution. If the mean and variance of binomial distribution are 4 and $\frac{4}{3}$, respectively, find $P(X < 1)$.

-OR -

Show that sum of independent normal variates is a normal variate.

4. In a Bernoulli distribution with parameter p, $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{2}{3}$ is rejected if more than 3 heads are obtained out of 5 throws of a coin. Find the probability of type –I and type-II errors.

-OR-

Define the following :

(i) Null and Alternative hypothesis
(ii) Type-I and Type-II errors
(iii) Size and power of the test